CHAPTER 6
PERCENTAGE AND MEASUREMENT

In the discussion of decimal fractions, it was shown that for convenience in writing fractions whose denominators are 10 or some power of 10, the decimal point could be employed and the denominators could be dropped. Thus, this special group of fractions could be written in a much simpler way. As early as the 15th century, businessmen made use of certain decimal fractions so much that they gave them the special designation PERCENT.

MEANING OF PERCENT

The word "percent" is derived from Latin. It was originally "per centum," which means "by the hundred." Thus the statement is often made that "percent means hundredths."

Percentage deals with the group of decimal fractions whose denominators are 100—that is, fractions of two decimal places. Since hundredths were used so frequently, the decimal point was dropped and the symbol % was placed after the number and read "percent" (per 100). Thus, 0.15 and 15% represent the same value, 15/100. The first is read "15 hundredths," and the second is read "15 percent." Both mean 15 parts out of 100.

Ordinarily, percent is used in discussing relative values. For example, 25 percent may convey an idea of relative value or relationship. To say "25 percent of the crew is ashore" gives an idea of what part of the crew is gone, but it does not tell how many. For example, 25 percent of the crew would represent vastly different numbers if the comparison were made between an LSM and a cruiser. When it is necessary to use a percent in computation, the number is written in its decimal form to avoid confusion.

By converting all decimal fractions so that they had the common denominator 100, men found that they could mentally visualize the relative size of the part of the whole that was being considered.

CHANGING DECIMALS TO PERCENT

Since percent means hundredths, any decimal may be changed to percent by first expressing it as a fraction with 100 as the denominator. The numerator of the fraction thus formed indicates how many hundredths we have, and therefore it indicates "how many percent" we have. For example, 0.36 is the same as 36/100. Therefore, 0.36 expressed as a percentage would be 36 percent. By the same reasoning, since 0.052 is equal to 5.2/100, 0.052 is the same as 5.2 percent.

In actual practice, the step in which the denominator 100 occurs is seldom written down. The expression in terms of hundredths is converted mentally to percent. This results in the following rule: To change a decimal to percent, multiply the decimal by 100 and annex the percent sign (%). Since multiplying by 100 has the effect of moving the decimal point two places to the right, the rule is sometimes stated as follows: To change a decimal to percent, move the decimal point two places to the right and annex the percent sign.

Changing Common Fractions and Whole Numbers To Percent

Common fractions are changed to percent by first expressing them as decimals. For example, the fraction 1/4 is equivalent to the decimal 0.25. Thus 1/4 is the same as 25 percent.

Whole numbers may be considered as special types of decimals (for example, 4 may be written as 4.00) and thus may be expressed in terms of percentage. The meaning of an expression such as 400 percent is vague unless we keep in mind that percentage is a form of comparison. For example, a question which often arises is "How can I have more than 100 percent of something, if 100 percent means all of it?"

This question seems reasonable, if we limit our attention to such quantities as test scores. However, it is also reasonable to use percentage in comparing a current set of data with a previous set. For example, if the amount of electrical power used by a Navy facility this year is double the amount used last year, then this year's power usage is 200 percent of last year's usage.

The meaning of a phrase such as "200 percent of last year's usage" is often misinterpreted. A total amount that is 200 percent of the previous amount is not the same as an increase of 200 percent. The increase in this case is only 100 percent, for a total of 200. If the increase had been 200 percent, then the new usage figure would be 300 percent of the previous figure.

Baseball batting averages comprise a special case in which percentage is used with only occasional reference to the word "percent." The percentages in batting averages are expressed in their decimal form, with the figure 1.000 representing 100 percent. Although a batting average of 0.300 is referred to as "batting 300," this is actually erroneous nomenclature from the strictly mathematical standpoint. The correct statement, mathematically, would be "batting point three zero zero" or "batting 30 percent."

Practice problems. Change each of the following numbers to percent:

1. 0.0065	3. 0.363	5. 7
2. 1.25	4. 3/4	6. 1/2

Answers:

1. 0.65%	3. 36.3%	5. 700%
2. 125%	4. 75%	6. 50%

CHANGING A PERCENT TO A DECIMAL

Since we do not compute with numbers in the percent form, it is often necessary to change a percent back to the decimal form. The procedure is just opposite to that used in changing decimals to percents: To change a percent to a decimal, drop the percent sign and divide the number by 100. Mechanically, the decimal point is simply shifted two places to the left and the percent sign is dropped. For example, 25 percent is the same as the decimal 0.25. Percents larger than 100 percent are changed to decimals by the same procedure as ordinary percents. For example, 125 percent is equivalent to 1.25.

Practice problems. Change the following percents to decimals:

1. 2.5%	3. 125%	5. $5\frac{3}{4}\%$
2. 0.63%	4. 25%	6. $9\frac{1}{2}\%$

Answers:

1. 0.025	3. 1.25	5. 5.75% = 0.0575
2. 0.0063	4. 0.25	6. 9.50% = 0.095

THE THREE PERCENTAGE CASES

To explain the cases that arise in problems involving percents, it is necessary to define the terms that will be used. Rate (r) is the number of hundredths parts taken. This is the number followed by the percent sign. The base (b) is the whole on which the rate operates. Percentage (p) is the part of the base determined by the rate. In the example

$$5\% \text{ of } 40 = 2$$

5% is the rate, 40 is the base, and 2 is the percentage.

There are three cases that usually arise in dealing with percentage, as follows:

Case I—To find the percentage when the base and rate are known.

EXAMPLE: What number is 6% of 50?

Case II—To find the rate when the base and percentage are known.

EXAMPLE: 20 is what percent of 60?

Case III—To find the base when the percentage and rate are known.

EXAMPLE: The number 5 is 25% of what number?

Case I

In the example

$$6\% \text{ of } 50 = ?$$

the "of" has the same meaning as it does in fractional examples, such as

$$\frac{1}{4} \text{ of } 16 = ?$$

In other words, "of" means to multiply. Thus, to find the percentage, multiply the base by the rate. Of course the rate must be changed from a percent to a decimal before multiplying can be done. Rate times base equals percentage. Thus,

$$6\% \text{ of } 50 = ?$$
$$0.06 \times 50 = 3$$

The number that is 6% of 50 is 3.

FRACTIONAL PERCENTS.—A fractional percent represents a part of 1 percent. In a case such as this, it is sometimes easier to find 1 percent of the number and then find the fractional part. For example, we would find 1/4 percent of 840 as follows:

$$\begin{aligned} 1\% \text{ of } 840 &= 0.01 \times 840 \\ &= 8.40 \end{aligned}$$

Therefore, $\frac{1}{4}\% \text{ of } 840 = 8.40 \times \frac{1}{4}$

$$= 2.10$$

Case II

To explain case II and case III, we notice in the foregoing example that the base corresponds to the multiplicand, the rate corresponds to the multiplier, and the percentage corresponds to the product.

$$\begin{array}{rl} 50 & \text{(base or multiplicand)} \\ \underline{.06} & \text{(rate or multiplier)} \\ 3.00 & \text{(percentage or product)} \end{array}$$

Recalling that the product divided by one of its factors gives the other factor, we can solve the following problem:

$$?\% \text{ of } 60 = 20$$

We are given the base (60) and percentage (20).

$$\begin{array}{rl} 60 & \text{(base)} \\ \underline{?} & \text{(rate)} \\ 20 & \text{(percentage)} \end{array}$$

We then divide the product (percentage) by the multiplicand (base) to get the other factor (rate). Percentage divided by base equals rate. The rate is found as follows:

$$\begin{aligned} \frac{20}{60} &= \frac{1}{3} \\ &= .33\frac{1}{3} \\ &= 33\frac{1}{3}\% \text{ (rate)} \end{aligned}$$

The rule for case II, as illustrated in the foregoing problem, is as follows: To find the rate when the percentage and base are known, divide the percentage by the base. Write the quotient in the decimal form first, and finally as a percent.

Case III

The unknown factor in case III is the base, and the rate and percentage are known.

EXAMPLE: $25\% \text{ of } ? = 5$

$$\begin{array}{rl} ? & \text{(base)} \\ \underline{.25} & \text{(rate)} \\ 5.00 & \text{(percentage)} \end{array}$$

We divide the product by its known factor to find the other factor. Percentage divided by rate equals base. Thus,

$$\frac{5}{.25} = 20 \text{ (base)}$$

The rule for case III may be stated as follows: To find the base when the rate and percentage are known, divide the percentage by the rate.

Practice problems. In each of the following problems, first determine which case is involved; then find the answer.

1. What is $\frac{3}{4}\%$ of 740?
2. 7.5% of 2.75 = ?
3. 8 is 2% of what number?
4. ?% of 18 = 15.
5. 12% of ? = 12.
6. 8 is what percent of 32?

Answers:

1. Case I; 5.55
2. Case I; 0.20625
3. Case III; 400
4. Case II; $83\frac{1}{3}\%$
5. Case III; 100
6. Case II; 25%

PRINCIPLES OF MEASUREMENT

Computation with decimals frequently involves the addition or subtraction of numbers which do not have the same number of decimal places. For example, we may be asked to add such numbers as 4.1 and 32.31582. How should they be added? Should zeros be annexed to 4.1 until it is of the same order as the other decimal (to the same number of places)? Or, should .31582 be rounded off to tenths? Would the sum be accurate to tenths or hundred-thousandths? The answers to these questions depend on how the numbers orignially arise.

Some decimals are finite or are considered as such because of their use. For instance, the decimal that represents $\frac{1}{2}$, that is 0.5, is as accurate at 0.5 as it is at 0.5000. Likewise, the decimal that represents $\frac{1}{8}$ has the value 0.125 and could be written just as accurately with additional end zeros. Such numbers are said to be finite. Counting numbers are finite. Dollars and cents are examples of finite values. Thus, $10.25 and $5.00 are finite values.

To add the decimals that represent $\frac{1}{8}$ and $\frac{1}{2}$, it is not necessary to round off 0.125 to tenths. Thus, 0.5 + 0.125 is added as follows:

$$\begin{array}{r} 0.500 \\ 0.125 \\ \hline 0.625 \end{array}$$

Notice that the end zeros were added to 0.5 to carry it out the same number of places as 0.125. It is not necessary to write such place-holding zeros if the figures are kept in the correct columns and decimal points are alined. Decimals that have a definite fixed value may be added or subtracted although they are of different order.

On the other hand, if the numbers result from measurement of some kind, then the question of how much to round off must be decided in terms of the precision and accuracy of the measurements.

ESTIMATION

Suppose that two numbers to be added resulted from measurement. Let us say that one number was measured with a ruler marked off in tenths of an inch and was found, to the nearest tenth of an inch, to be 2.3 inches. The other number measured with a precision rule was found, to the nearest thousandth of an inch, to be 1.426 inches.

Each of these measurements requires estimation between marks on the rule, and estimation between marks on any measuring instrument is subject to human error. Experience has shown that the best the average person can do with consistency is to decide whether a measurement is more or less than halfway between marks. The correct way to state this fact mathematically is to say that a measurement made with an instrument marked off in tenths of an inch involves a maximum probable error of 0.05 inch (five hundredths is one-half of one tenth). By the same reasoning, the probable error in a measurement made with an instrument marked in thousandths of an inch is 0.0005 inch.

PRECISION

In general, the probable error in any measurement is one-half the size of the smallest division on the measuring instrument. Thus the precision of a measurement depends upon how precisely the instrument is marked. It is important to realize that precision refers to the size of the smallest division on the scale; it has nothing to do with the correctness of the markings. In other words, to say that one instrument is more *precise* than *another* does not *imply* that the less *precise* instrument is poorly manufactured. In fact, it would be possible to make an instrument with very high apparent precision, and yet mark it carelessly so that measurements taken with it would be inaccurate.

From the mathematical standpoint, the precision of a number resulting from measurement depends upon the number of decimal places; that is, a larger number of decimal places means a smaller probable error. In 2.3 inches the probable error is 0.05 inch, since 2.3 actually lies somewhere between 2.25 and 2.35. In 1.426 inches there is a much smaller probable error of 0.0005 inch. If we add 2.300 + 1.426 and get an answer in thousandths, the answer, 3.726 inches, would appear to be precise to thousandths; but this is not true since there was a probable error of .05 in one of the addends. Also 2.300 appears to be precise to thousandths but in this example it is precise only to tenths. It is evident that the precision of a sum is no greater than the precision of the least precise addend. It can also be shown that the precision of a difference is no greater than the less precise number compared.

To add or subtract numbers of different orders, all numbers should first be rounded off to the order of the least precise number. In the foregoing example, 1.426 should be rounded to tenths—that is, 1.4.

This rule also applies to repeating decimals. Since it is possible to round off a repeating decimal at any desired point, the degree of precision desired should be determined and all repeating decimals to be added should be rounded to this level. Thus, to add the decimals generated by $\frac{1}{3}$, $\frac{2}{3}$, and $\frac{5}{12}$ correct to thousandths, first round off each decimal to thousandths, and then add, as follows:

$$\begin{array}{r} .333 \\ .667 \\ \underline{.417} \\ 1.417 \end{array}$$

When a common fraction is used in recording the results of measurement, the denominator of the fraction indicates the degree of precision. For example, a ruler marked in sixty-fourths of an inch has smaller divisions than one marked in sixteenths of an inch. Therefore a measurement of $3\frac{4}{64}$ inches is more precise than a measure of $3\frac{1}{16}$ inches, even though the two fractions are numerically equal. Remember that a measurement of $3\frac{4}{64}$ inches contains a probable error of only one-half of one sixty-fourth of an inch. On the other hand, if the smallest division on the ruler is one-sixteenth of an inch, then a measurement of $3\frac{1}{16}$ inches contains a probable error of one thirty-second of an inch.

ACCURACY

Even though a number may be very precise, which indicates that it was measured with an instrument having closely spaced divisions, it may not be very accurate. The accuracy of a measurement depends upon the relative size of the probable error when compared with the quantity being measured. For example, a distance of 25 yards on a pistol range may be measured carefully enough to be correct to the nearest inch. Since there are 900 inches in 25 yards, this measurement is between 899.5 inches and 900.5 inches. When compared with the total of 900 inches, the 0.5-inch probable error is not very great.

On the other hand, a length of pipe may be measured rather precisely and found to be 3.2 inches long. The probable error here is 0.05 inch, and this measurement is thus more precise than that of the pistol range mentioned before. To compare the accuracy of the two measurements, we note that 0.05 inch out of a total of 3.2 inches is the same as 0.5 inch out of 32 inches. Comparing this with the figure obtained in the other example (0.5 inch out of 900), we conclude that the more precise measurement is actually the less accurate of the two measurements considered.

It is important to realize that the location of the decimal point has no bearing on the accuracy of the number. For example, 1.25 dollars represents exactly the same amount of money as 125 cents. These are equally accurate ways of representing the same quantity, despite the fact that the decimal point is placed differently.

Practice problems. In each of the following problems, determine which number of each pair is more accurate and which is more precise:

1. 3.72 inches or 2,417 feet
2. 2.5 inches or 17.5 inches
3. $5\frac{3}{4}$ inches or $12\frac{7}{8}$ inches
4. 34.2 seconds or 13 seconds

Answers:

1. 3.72 inches is more precise.
   2,417 feet is more accurate.
2. The numbers are equally precise.
   17.5 inches is more accurate.
3. $12\frac{7}{8}$ inches is more precise and more accurate.
4. 34.2 seconds is more precise and more accurate.

Percent of Error

The accuracy of a measurement is determined by the RELATIVE ERROR. The relative error is the ratio between the probable error and the quantity being measured. This ratio is simply the fraction formed by using the probable error as the numerator and the measurement itself as the denominator. For example, suppose that a metal plate is found to be 5.4 inches long, correct to the nearest tenth of an inch. The maximum probable error is five hundredths of an inch (one-half of one tenth of an inch) and the relative error is found as follows:

$$\frac{\text{probable error}}{\text{measured value}} = \frac{0.05}{5.4}$$
$$= \frac{5}{540}$$

Thus the relative error is 5 parts out of 540.

Relative error is usually expressed as PERCENT OF ERROR. When the denominator of the fraction expressing the error ratio is divided into the numerator, a decimal is obtained. This decimal, converted to percent, gives the percent of error. For example, the error in the foregoing problem could be stated as 0.93 percent, since the ratio 5/540 reduces to 0.0093 (rounded off) in decimal form.

Significant Digits

The accuracy of a measurement is often described in terms of the number of significant digits used in expressing it. If the digits of a number resulting from measurement are examined one by one, beginning with the left-hand digit, the first digit that is not 0 is the first significant digit. For example, 2345 has four significant digits and 0.023 has only two significant digits.

The digits 2 and 3 in a measurement such as 0.023 inch signify how many thousandths of an inch comprise the measurement. The 0's are of no significance in specifying the number of thousandths in the measurement; their presence is required only as "place holders" in placing the decimal point.

A rule that is often used states that the significant digits in a number begin with the first nonzero digit (counting from left to right) and end with the last digit. This implies that 0 can be a significant digit if it is not the first digit in the number. For example, 0.205 inch is a measurement having three significant digits. The 0 between the 2 and the 5 is significant because it is a part of the number specifying how many hundredths are in the measurement.

The rule stated in the foregoing paragraph fails to classify final 0's on the right. For example, in a number such as 4,700, the number of significant digits might be two, three, or four. If the 0's merely locate the decimal point (that is, if they show the number to be approximately forty-seven hundred rather than forty seven), then the number of significant digits is two. However, if the number 4,700 represents a number such as 4,730 rounded off to the nearest hundred, there are three significant digits. The last 0 merely locates the decimal point. If the number 4,700 represents a number such as 4,700.4 rounded off, then the number of significant digits is four.

Unless we know how a particular number was measured, it is sometimes impossible to determine whether right-hand 0's are the result of rounding off. However, in a practical situation it is normally possible to obtain information concerning the instruments used and the degree of precision of the original data before any rounding was done.

In a number such as 49.30 inches, it is reasonable to assume that the 0 in the hundredths place would not have been recorded at all if it were not significant. In other words, the instrument used for the measurement can be read to the nearest hundredth of an inch. The 0 on the right is thus significant. This conclusion can be reached another way by observing that the 0 in 49.30 is not needed as a place holder in placing the decimal point. Therefore its presence must have some other significance.

The facts concerning significant digits may be summarized as follows:

1. Digits other than 0 are always significant.
2. Zero is significant when it falls between significant digits.
3. Any final 0 to the right of the decimal point is significant.
4. When a 0 is present only as a place holder for locating the decimal point, it is not significant.
5. The following categories comprise the significant digits of any measurement number:

    a. The first nonzero left-hand digit is significant.

    b. The digit which indicates the precision of the number is significant. This is the digit farthest to the right, except when the right-hand digit is 0. If it is 0, it may be only a place holder when the number is an integer.

c. All digits between significant digits are significant.

Practice problems. Determine the percent of error and the number of significant digits in each of the following measurements:

1. 5.4 feet
2. 0.00042 inch
3. 4.17 sec
4. 147.50 miles

Answers:

1. Percent of error: 0.93%
   Significant digits: 2
2. Percent of error: 1.19%
   Significant digits: 2
3. Percent of error: 0.12%
   Significant digits: 3
4. Percent of error: 0.0034%
   Significant digits: 5

CALCULATING WITH APPROXIMATE NUMBERS

The concepts of precision and accuracy form the basis for the rules which govern calculation with approximate numbers (numbers resulting from measurement).

Addition and Subtraction

A sum or difference can never be more precise than the least precise number in the calculation. Therefore, before adding or subtracting approximate numbers, they should be rounded to the same degree of precision. The more precise numbers are all rounded to the precision of the least precise number in the group to be combined. For example, the numbers 2.95, 32.7, and 1.414 would be rounded to tenths before adding as follows:

$$\begin{array}{r} 3.0 \\ 32.7 \\ \underline{1.4} \end{array}$$

Multiplication and Division

When two numbers are multiplied, the result often has several more digits than either of the original factors. Division also frequently produces more digits in the quotient than the original data possessed, if the division is "carried out" to several decimal places. Results such as these appear to have more significant digits than the original measurements from which they came, giving the false impression of greater accuracy than is justified. In order to correct this situation, the following rule is used:

In order to multiply or divide two approximate numbers having an equal number of significant digits, round the answer to the same number of significant digits as are shown in one of the original numbers. If one of the original factors has more significant digits than the other, round the more accurate number before multiplying. It should be rounded to one more significant digit than appears in the less accurate number; the extra digit protects the answer from the effects of multiple rounding. After performing the multiplication or division, round the result to the same number of significant digits as are shown in the less accurate of the original factors.

Practice problems:

1. Find the sum of the sides of a triangle in which the lengths of the three sides are as follows: 2.5 inches, 3.72 inches, and 4.996 inches.
2. Find the product of the length and width of a rectangle which is 2.95 feet long and 0.9046 foot wide.

Answers:

1. 11.2 inches
2. 2.67 square feet

MICROMETERS AND VERNIERS

Closely associated with the study of decimals is a measuring instrument known as a micrometer. The ordinary micrometer is capable of measuring accurately to one-thousandth of an inch. One-thousandth of an inch is about the thickness of a human hair or a thin sheet of paper. The parts of a micrometer are shown in figure 6-1.

MICROMETER SCALES

The spindle and the thimble move together. The end of the spindle (hidden from view in figure 6-1) is a screw with 40 threads per inch. Consequently, one complete turn of the thimble moves the spindle one-fortieth of an inch or

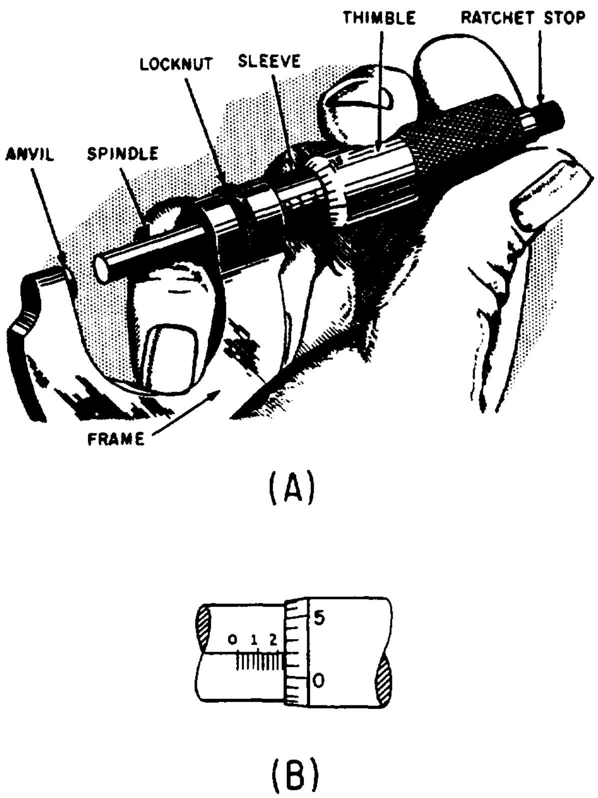

Figure 6-1.–(A) Parts of a micrometer; (B) micrometer scales.

0.025 inch since $\frac{1}{40}$ is equal to 0.025. The sleeve has 40 markings to the inch. Thus each space between the markings on the sleeve is also 0.025 inch. Since 4 such spaces are 0.1 inch (that is, 4 x 0.025), every fourth mark is labeled in tenths of an inch for convenience in reading. Thus, 4 marks equal 0.1 inch, 8 marks equal 0.2 inch, 12 marks equal 0.3 inch, etc.

To enable measurement of a partial turn, the beveled edge of the thimble is divided into 25 equal parts. Thus each marking on the thimble is $\frac{1}{25}$ of a complete turn, or $\frac{1}{25}$ of $\frac{1}{40}$ of an inch. Multiplying $\frac{1}{25}$ times 0.025 inch, we find that each marking on the thimble represents 0.001 inch.

READING THE MICROMETER

It is sometimes convenient when learning to read a micrometer to writedown the component parts of the measurement as read on the scales and then to add them. For example, in figure 6-1 (B) there are two major divisions visible (0.2 inch). One minor division is showing clearly (0.025 inch). The marking on the thimble nearest the horizontal or index line of the sleeve is the second marking (0.002 inch). Adding these parts, we have

$$\begin{array}{r} 0.200 \\ 0.025 \\ 0.002 \\ \hline 0.227 \end{array}$$

Thus, the reading is 0.227 inch. As explained previously, this is read verbally as "two hundred twenty-seven thousandths." A more skillful method of reading the scales is to read all digits as thousandths directly and to do any adding mentally. Thus, we read the major division on the scale as “two hundred thousandths” and the minor division is added on mentally. The mental process for the above setting then would be “two hundred twenty-five; two hundred twenty-seven thousandths.”

Practice problems:

1. Read each of the micrometer settings shown in figure 6-2.

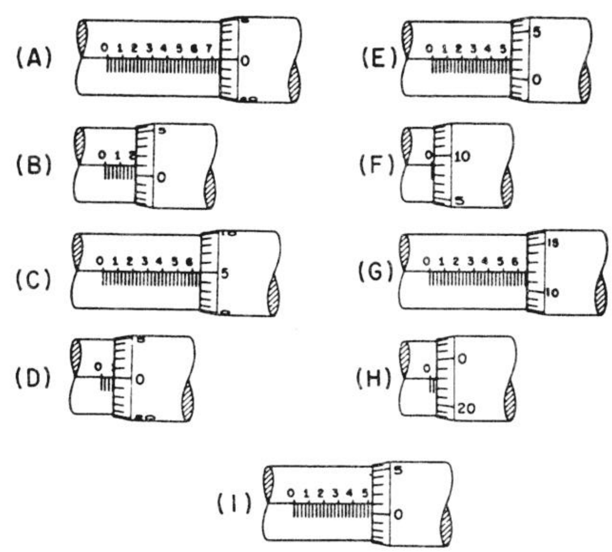

Figure 6-2.–Micrometer settings.

Answers:

1. (A) 0.750 (F) 0.009
   (B) 0.201 (G) 0.662
   (C) 0.655 (H) 0.048
   (D) 0.075 (I) 0.526
   (E) 0.527

VERNIER

Sometimes the marking on the thimble of the micrometer does not fall directly on the index line of the sleeve. To make possible readings even smaller than thousandths, an ingenious device is introduced in the form of an additional scale. This scale, called a VERNIER, was named after its inventor, Pierre Vernier. The vernier makes possible accurate readings to the ten-thousandth of an inch.

Principle of the Vernier

Suppose a ruler has markings every tenth of an inch but it is desired to read accurately to hundredths. A separate, freely sliding vernier scale (fig. 6-3) is added to the ruler. It has 10 markings on it that take up the same distance as 9 markings on the ruler scale. Thus, each space on the vernier is $\frac{1}{10}$ of $\frac{9}{10}$ inch, or $\frac{9}{100}$ inch. How much smaller is a space on the vernier than a space on the ruler? The ruler space is $\frac{1}{10}$ inch, or $\frac{10}{100}$ inch, and the vernier space is $\frac{9}{100}$ inch. The vernier space is smaller by the difference between these two numbers, as follows:

$$\frac{10}{100} - \frac{9}{100} = \frac{1}{100}$$

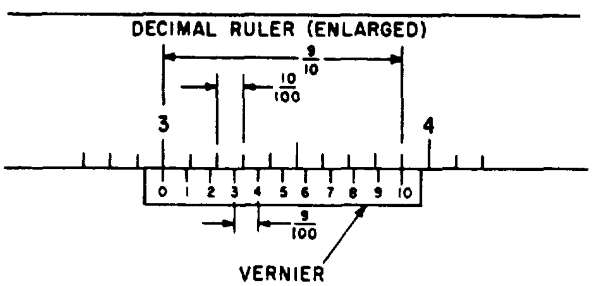

Figure 6-3.—Vernier scale.

Each vernier space is $\frac{1}{100}$ inch smaller than a ruler space.

As an example of the use of the vernier scale, suppose that we are measuring the steel bar shown in figure 6-4. The end of the bar almost reaches the 3-inch mark on the ruler, and we estimate that it is about halfway between 2.9 inches and 3.0 inches. The vernier marks help us to decide whether the exact measurement is 2.94 inches, 2.95 inches, or 2.96 inches.

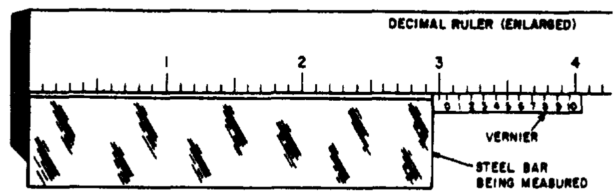

Figure 6-4.—Measuring with a vernier.

The 0 on the vernier scale is spaced the distance of exactly one ruler mark (in this case, one tenth of an inch) from the left hand end of the vernier. Therefore the 0 is at a position between ruler marks which is comparable to the position of the end of the bar. In other words, the 0 on the vernier is about halfway between two adjacent marks on the ruler, just as the end of the bar is about halfway between two adjacent marks. The 1 on the vernier scale is a little closer to alinement with an adjacent ruler mark; in fact, it is one hundredth of an inch closer to alinement than the 0. This is because each space on the vernier is one hundredth of an inch shorter than each space on the ruler.

Each successive mark on the vernier scale is one hundredth of an inch closer to alinement than the preceding mark, until finally alinement is achieved at the 5 mark. This means that the 0 on the vernier must be five hundredths of an inch from the nearest ruler mark, since five increments, each one hundredth of an inch in size, were used before a mark was found in alinement.

We conclude that the end of the bar is five hundredths of an inch from the 2.9 mark on the ruler, since its position between marks is exactly comparable to that of the 0 on the vernier scale. Thus the value of our measurement is 2.95 inches.

The foregoing example could be followed through for any distance between markings. Suppose the 0 mark fell seven tenths of the distance between ruler markings. It would take seven vernier markings, a loss of one-hundredth of an inch each time, to bring the marks in line at 7 on the vernier.

The vernier principle may be used to get fine linear readings, angular readings, etc. The principle is always the same. The vernier has one more marking than the number of markings on an equal space of the conventional scale of the measuring instrument. For example, the vernier caliper (fig. 6-5) has 25 markings on the vernier for 24 on the caliper scale. The caliper is marked off to read to fortieths (0.025) of an inch, and the vernier extends the accuracy to a thousandth of an inch.

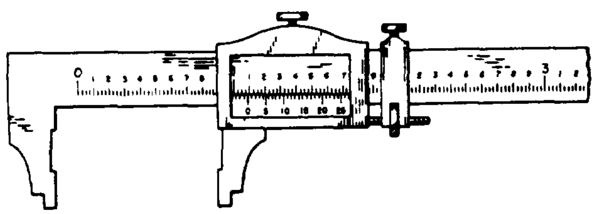

Figure 6-5.–A vernier caliper.

Vernier Micrometer

By adding a vernier to the micrometer, it is possible to read accurately to one ten-thousandth of an inch. The vernier markings are on the sleeve of the micrometer and are parallel to the thimble markings. There are 10 divisions on the vernier that occupy the same space as 9 divisions on the thimble. Since a thimble space is one thousandth of an inch, a vernier space is $\frac{1}{10}$ of $\frac{9}{1000}$ inch, or $\frac{9}{10000}$ inch. It is $\frac{1}{10000}$ inch less than a thimble space. Thus, as in the preceding explanation of verniers, it is possible to read the nearest ten-thousandth of an inch by reading the vernier digit whose marking coincides with a thimble marking.

In figure 6-6 (A), the last major division showing fully on the sleeve index is 3. The third minor division is the last mark clearly showing (0.075). The thimble division nearest and below the index is the 8 (0.008). The vernier marking that matches a thimble marking is the fourth (0.0004). Adding them all together, we have,

$$\begin{array}{r} 0.3000 \\ 0.0750 \\ 0.0080 \\ \underline{0.0004} \\ 0.3834 \end{array}$$

The reading is 0.3834 inch. With practice these readings can be made directly from the micrometer, without writing the partial readings.

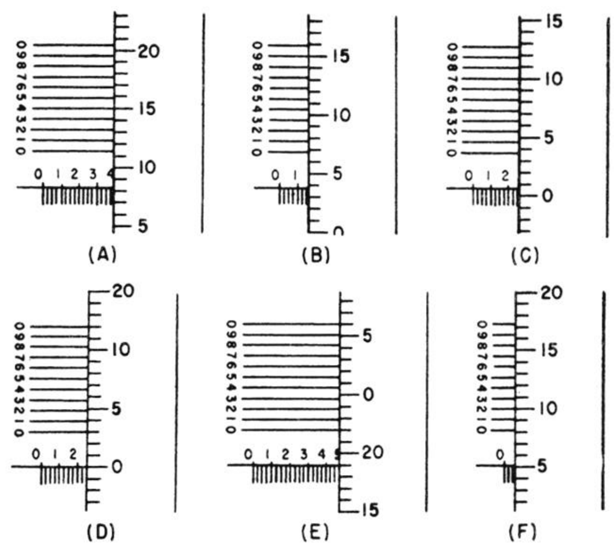

Figure 6-6.–Vernier micrometer settings.

Practice problems:

1. Read the micrometer settings in figure 6-6.

Answers:

1. (A) See the foregoing example.
   (B) 0.1539
   (C) 0.2507
   (D) 0.2500
   (E) 0.4690
   (F) 0.0552